Paper 4A: Further Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(i)	$602 = 3\times161+119$	M1	1.1b
	$161 = 119 + 42,\ 119 = 2\times 42 + 35$	M1	1.1b
	$42 = 35 + 7,\ 35 = 5\times 7,\ \text{hcf} = 7$	A1	1.1b
		(3)	
(ii)	Number of codes under old system = $5\times4\times4\times3\times2\ (= 480)$	B1	3.1b
	Number of codes under new system = $4\times3\times7\times6\times5\ (= 2520)$	B1	3.1b
	Subtracts first answer from second	M1	1.1b
	Increase in number of codes is 2040	A1	1.1b
		(4)	
			(7 marks)

Notes:

(i)

M1: Attempts Euclid's algorithm – (there may be an arithmetic slip finding 119)

M1: Uses Euclid's algorithm a further two times with 161 and "their 119" and then with "their 119" and "their 42"

A1: This should be accurate with all the steps shown

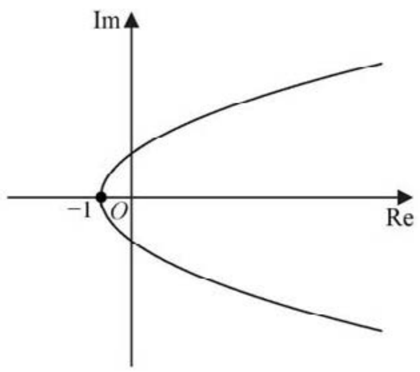
(ii)

B1: Correctly interprets the problem and uses the five odd digits and four even digits to form a correct product

B1: Interprets the new situation using the four even digits, then the seven digits which have not been used, to form a correct product

M1: Subtracts one answer from the other

A1: Correct answer

Question	Scheme	Marks	AOs
2(a)	Let $z = x + \mathrm{i}$	M1	2.1
	$w=(x+\mathrm{i})^2=(x^2-1)+2x\mathrm{i}$	A1	1.1b
	Let $w = u + \mathrm{i}v$, then $u=(x^2-1)$ and $v=2x$	M1	2.1
	$\Rightarrow v^2=4(u+1)$, which therefore represents a parabola	A1ft	2.2a
		(4)	
(b)	 M1: Sketches a parabola with symmetry about the real axis A1: Accurate sketch	M1 A1	1.1b 1.1b
		(2)	
			(6 marks)

Notes:

(a)

M1: Translates the information that Im(z) = 1 into a cartesian form; e.g. $z = x + \mathrm{i}$

A1: Obtains a correct expression for w

M1: Separates the real and imaginary parts and equates to u and v respectively

A1ft: Obtains a quadratic equation and states that their quadratic equation represents a parabola

(b)

M1: Sketches a parabola with symmetry about the real axis

A1: Accurate sketch

Question	Scheme		Marks	AOs
3(a)	Finds the characteristic equation $(2-\lambda)^2(4-\lambda)-(4-\lambda)=0$		M1	2.1
	So $(4-\lambda)(\lambda^2-4\lambda+3)=0$ so $\lambda=4$ *		A1*	2.2a
	Solves quadratic equation to give		M1	1.1b
	$\lambda=1$ and $\lambda=3$		A1	1.1b
			(4)	
(b)	Uses a correct method to find an eigenvector		M1	1.1b
	Obtains a vector parallel to one of $\begin{pmatrix}0\\0\\1\end{pmatrix}$ or $\begin{pmatrix}1\\1\\1\end{pmatrix}$ or $\begin{pmatrix}3\\-3\\1\end{pmatrix}$		A1	1.1b
	Obtains two correct vectors		A1	1.1b
	Obtains all three correct vectors		A1	1.1b
			(4)	
(c)	Uses their three vectors to form a matrix		M1	1.2
	$\begin{pmatrix}0&1&3\\0&1&-3\\1&1&1\end{pmatrix}$	**or** other correct answer with columns in a different order	A1	1.1b
			(2)	
			(10 marks)	

Notes:

(a)

M1: Attempts to find the characteristic equation (there may be one slip)

A1*: Deduces that $\lambda=4$ is a solution by the method shown or by checking that $\lambda=4$ satisfies the characteristic equation

M1: Solves their quadratic equation

A1: Obtains the two correct answers as shown above

(b)

M1: Uses a correct method to find an eigenvector

A1: Obtains one correct vector (may be a multiple of the given vectors)

A1: Obtains two correct vectors (may be multiples of the given vectors)

A1: Obtains all three correct vectors (may be multiples of the given vectors)

(c)

M1: Forms a matrix with their vectors as columns

A1: $\begin{pmatrix}0&1&3\\0&1&-3\\1&1&1\end{pmatrix}$ **or** $\begin{pmatrix}1&0&3\\1&0&-3\\1&1&1\end{pmatrix}$ **or** $\begin{pmatrix}3&1&0\\-3&1&0\\1&1&1\end{pmatrix}$ **or** other correct alternative

Question	Scheme	Marks	AOs
4(i)	If we assume $ab = ba$; as $a^2b = ba$ then $ab = a^2b$	M1	2.1
	So $a^{-1}abb^{-1} = a^{-1}a^2bb^{-1}$	M1	2.1
	So $e = a$	A1	2.2a
	But this is a contradiction, as the elements e and a are distinct so $ab \neq ba$	A1	2.4
		(4)	
(ii)(a)	2 has order 4 and 4 has order 2	M1	1.1b
	7, 8 and 13 have order 4	A1	1.1b
	11 and 14 have order 2 and 1 has order 1	A1	1.1b
		(3)	
(ii)(b)	Finds the subgroup {1, 2, 4, 8} or the subgroup {1, 7, 4, 13}	M1	1.1b
	Finds both and refers to them as cyclic groups, or gives generator 2 and generator 7	A1	2.4
	Finds {1, 4, 11, 14}	B1	2.2a
	States each element has order 2 or refers to it as Klein Group	B1	2.5
		(4)	
(ii)(c)	*J* has an element of order 8, (*H* does not) or *J* is a cyclic group (*H* is not) or other valid reason	M1	2.4
	They are not isomorphic	A1	2.2a
		(2)	
			(13 marks)

Question 4 notes:	
(i)	
M1:	Proof begins with assumption that $ab = ba$ and deduces that this implies $ab = a^2b$
M1:	A correct proof with working shown follows, and may be done in two stages
A1:	Concludes that assumption implies that $e = a$
A1:	Explains clearly that this is a contradiction, as the elements *e* and *a* are distinct so $ab \neq ba$
(ii)(a)	
M1:	Obtains two correct orders (usually the two in the scheme)
A1:	Finds another three correctly
A1:	Finds the final three so that all eight are correct
(ii)(b)	
M1:	Finds one of the cyclic subgroups
A1:	Finds both subgroups and explains that they are cyclic groups, or gives generators 2 and 7
B1:	Finds the non cyclic group
B1:	Uses correct terms that each element has order 2 or refers to it as Klein Group
(ii)(c)	
M1:	Clearly explains how *J* differs from *H*
A1:	Correct deduction

Question	Scheme	Marks	AOs
5(a)	$\frac{dy}{dx} = -\sinh 2x$	B1	2.1
	So $S = \int \sqrt{1+\sinh^2 2x}\,dx$	M1	2.1
	$\therefore\ s = \int \cosh 2x\,dx$	A1	1.1b
	$= \left[\frac{1}{2}\sinh 2x\right]_{-\ln a}^{\ln a}$ or $\left[\sinh 2x\right]_0^{\ln a}$	M1	2.1
	$= \sinh 2\ln a = \frac{1}{2}[e^{2\ln a} - e^{-2\ln a}] = \frac{1}{2}\left(a^2 - \frac{1}{a^2}\right)$ (so $k = ½$)	A1	1.1b
		(5)	
(b)	$\frac{1}{2}\left(a^2 - \frac{1}{a^2}\right) = 2$ so $a^4 - 4a^2 - 1 = 0$	M1	1.1b
	$a^2 = 2 + \sqrt{5}$ (and a = 2.06 (approx.))	M1	1.1b
	When $x = \ln a$, $y = 0$ so $A = \frac{1}{2}\cosh(2\ln a)$	M1	3.4
	Height = A – 0.5 = awrt 0.62m	A1	1.1b
		(4)	
(c)	The width of the base = 2ln$a$ = 1.4m	B1	3.4
		(1)	
(d)	A parabola of the form $y = 0.62 - 1.19\,x^2$, or other symmetric curve with its equation e.g. $0.62\cos(2.2x)$	M1A1	3.3 3.3
		(2)	
			(12 marks)

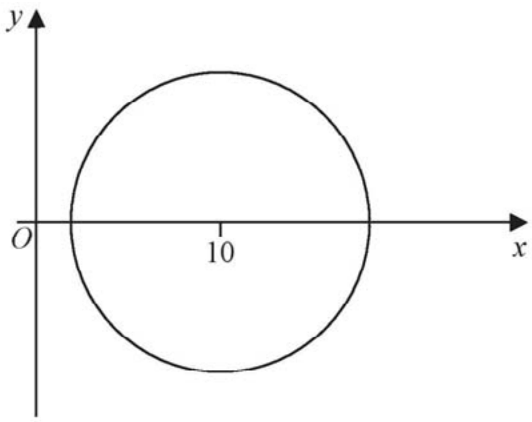
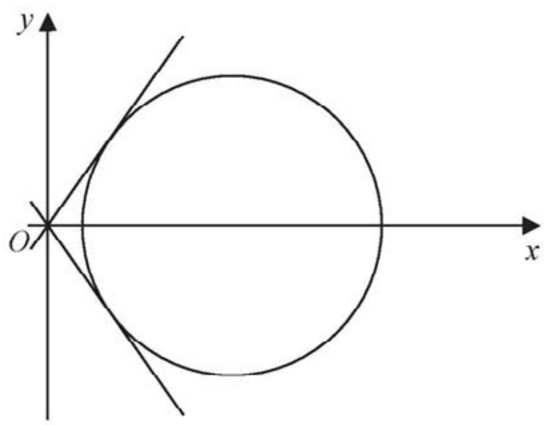
Notes:

(a)
B1: Starts explanation by finding the correct derivative
M1: Uses their derivative in the formula for arc length
A1: Uses suitable identity to simplify the integrand and to obtain the expression in scheme
M1: Integrates and uses appropriate limits to find the required arc length
A1: Uses the definition of sinh to complete the proof and identifies the value for k

(b)
M1: Uses the formula obtained from the model and the length of the arch to create a quartic equation
M1: Continues to use this model to obtain a quadratic and to obtain values for a
M1: Attempts to find a value for A in order to find h
A1: Finds a value for the height correct to 2sf (or accept exact answer)

(c)
B1: Finds width to 2 sf i.e. 1.4m

(d)
M1: Chooses or describes an even function with maximum point on the y axis
A1: Gives suitable equation passing through (0, 0.62) and (0.7, 0) and (- 0.7, 0)

Question	Scheme	Marks	AOs
6(a)	$(x+6)^2 + y^2 = 4[(x-6)^{+}y^2]$	M1	2.1
	$x^2 + y^2 - 20x + 36 = 0$ which is the equation of a circle*	A1*	2.2a
		(2)	
(b)		M1	1.1b
		A1	1.1b
		(2)	
(c)	Let $a = c + \mathrm{i}d$ and $a^* = c - \mathrm{i}d$ then $(c + \mathrm{i}d)(x - \mathrm{i}y) + (c - \mathrm{i}d)(x + \mathrm{i}y) = 0$	M1	3.1a
	So $y = -\frac{c}{d}x$	A1	1.1b
	 The gradients of the tangents (from geometry) are $\pm\frac{4}{3}$	B1	3.1a
	So $-\frac{c}{d} = \pm\frac{4}{3}$ and $\frac{d}{c} = \mp\frac{3}{4}$	M1	3.1a
	So $\tan\theta = \pm\frac{3}{4}$	A1	1.1b
		(5)	

Question 6 notes:	
(a)	
M1:	Obtains an equation in terms of x and y using the given information
A1*:	Expands and simplifies the algebra, collecting terms and obtains a circle equation correctly, deducing that this is a circle
(b)	
M1:	Draws a circle with centre at (10, 0)
A1:	(Radius is 8) so circle does not cross the y axis
(c)	
M1:	Attempts to convert line equation into a cartesian form
A1:	Obtains a simplified line equation
B1:	Uses geometry to deduce the gradients of the tangents
M1:	Understands the connection between arg a and the gradient of the tangents and uses this connection
A1:	Correct answers

Question	Scheme	Marks	AOs
7(a)	$I_n = \int_0^{\frac{\pi}{2}} \sin x \sin^{n-1} x \, dx$	M1	2.1
	$= \left[-\cos x \sin^{n-1} x\right]_0^{\frac{\pi}{2}} - (-)\int_0^{\frac{\pi}{2}} \cos^2 x(n-1)\sin^{n-2} x \, dx$	A1	1.1b
	Obtains $= 0 - (-)\int_0^{\frac{\pi}{2}} (1-\sin^2 x)(n-1)\sin^{n-2} x \, dx$	M1	1.1b
	So $I_n = (n-1)I_{n-2} - (n-1)I_n$ and hence $nI_n = (n-1)I_{n-2}$ *	A1*	2.1
		(4)	
(b)	uses $I_n = \frac{(n-1)}{n} I_{n-2}$ to give $I_{10} = \frac{9}{10} I_8$ or $I_2 = \frac{1}{2} I_0$	M1	3.1b
	So $I_{10} = \frac{9\times7\times5\times3\times1}{10\times8\times6\times4\times2} I_0$ x	M1	2.1
	$I_0 = \frac{\pi}{2}$	B1	1.1b
	Required area is $2(I_2 - I_{10}) =$ **or** $8\times\frac{1}{4}(I_2 - I_{10}) =$	M1	3.1b
	$= 2\left(\frac{\pi}{4} - \frac{63\pi}{512}\right) = \frac{65\pi}{256}$ m^2	A1	1.1b
		(5)	
			(9 marks)

Notes:

(a)

M1: Splits the integrand into the product shown and begins process of integration by parts (there may be sign errors)

A1: Correct work

M1: Uses limits on the first term and expresses $\cos^2$ term in terms of $\sin^2$

A1*: Completes the proof collecting I_n terms correctly with all stages shown

(b)

M1: Attempts to find I_{10} and/or I_2

M1: Finds I_{10} in terms of I_0

B1: Finds I_0 correctly

M1: States the expression needed to find the required area

A1: Completes the calculation to give this exact answer

Question	Scheme	Marks	AOs
8(a)	$u_1 = 1$ as there is only one way to go up one step	B1	2.4
	$u_2 = 2$ as there are two ways: one step then one step or two steps	B1	2.4
	If first move is one step then can climb the other (n–1) steps in u_{n-1} ways. If first move is two steps can climb the other (n–2) steps in u_{n-2} ways so $u_n = u_{n-1} + u_{n-2}$	B1	2.4
		(3)	
(b)	Sequence begins 1, 2, 3, 5, 8, 13, 21, 34,… so 34 ways of climbing 8 steps	B1	1.1b
		(1)	
(c)	To find general term use $u_n = u_{n-1} + u_{n-2}$ gives $\lambda^2 = \lambda + 1$	M1	2.1
	This has roots $\frac{1 \pm \sqrt{5}}{2}$	A1	1.1b
	So general form is $A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$	M1	2.2a
	Uses initial conditions to find A and B reaching two equations in A and B	M1	1.1b
	Obtains $A = \left(\frac{1+\sqrt{5}}{2\sqrt{5}}\right)$ and $B = -\left(\frac{1-\sqrt{5}}{2\sqrt{5}}\right)$ and so when n = 400 obtains $\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{401} - \left(\frac{1-\sqrt{5}}{2}\right)^{401}\right]$ *	A1*	1.1b
		(5)	
	(9 marks)		

Notes:

(a)

B1: Need to see explanation for $u_1 = 1$

B1: Need to see explanation for $u_2 = 2$ with the two ways spelled out

B1: Need to see the first move can be one step or can be two steps and clear explanation of the iterative expression as in the scheme

(b)

B1: The answer is enough for this mark

(c)

M1: Obtains this characteristic equation

A1: Solves quadratic – giving exact answers

M1: Obtains a general form

M1: Use initial conditions to obtains two equations which should be $A\left(1+\sqrt{5}\right) + B\left(1-\sqrt{5}\right) = 2$ o.e. and $A\left(3+\sqrt{5}\right) + B\left(3-\sqrt{5}\right) = 4$ but allow slips here

A1*: Must see exact correct values for A and B and conclusion given for n = 400

Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number	Candidate Number

Further Mathematics

Advanced
Further Mathematics Option 1
Paper 3: Further Statistics 1
Further Mathematics Option 2
Paper 4: Further Statistics 1

Sample Assessment Material for first teaching September 2017	Paper Reference
Time: 1 hour 30 minutes	**9FM0/3B** **9FM0/4B**

You must have: Mathematical Formulae and Statistical Tables, calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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